The design variables are the length (a'') and the thickness (h'') of the plate. The objective function to be minimized is given by $f(a,h) = 0.0787\ a^2h$ lbs. The damping ratio corresponding to the fundamental mode is assumed as $\zeta = 0.01$, and the upper and lower bounds on the fundamental frequency are taken as 40.0 and 100.0 rad/sec, respectively. In addition, the design variables are restricted as $50'' \leq a \leq 100''$ and $0.25'' \leq h \leq 1.50''$.

In this case, the fundamental mode shape is given by[5]

$$\phi_{11}(x,y) = \sin\frac{\pi x}{a} \cdot \sin\frac{\pi y}{b} \tag{17}$$

and the corresponding frequency by

$$\omega_{11} = 2\pi f = \pi^2\sqrt{Eh^3/12\mu(1-\nu^2)}\left[\frac{1}{a^2}+\frac{1}{b^2}\right] \tag{18}$$

For a uniform load $F(t)$ acting on the plate, the modal force, $G(t)$, is given by

$$G(t) = 4ab/\pi^2 \cdot F(t) \tag{19}$$

The modal mass and modal stiffness are given by

$$M = \mu S \sin \pi x/a \cdot \sin \pi y/b \tag{20}$$

and

$$K = \omega_{11}^2 \cdot M \tag{21}$$

The maximum stress (s) induced under the maximum permissible central deflection Z is given by[6]

$$s = \frac{\pi^2 EhZ}{2(1-\nu^2)}\left[\frac{\nu}{a^2}+\frac{1}{b^2}\right] \tag{22}$$

Here E denotes the Young's modulus, μ the mass per unit area, ν the Poisson's ratio, and S the surface area of the plate.

Once the plate is modelled as a single degree of freedom oscillator, the root mean square displacement of the oscillator, $z_{\rm rms}$, under the excitation of a Gaussian noise can be expressed as[7]

$$z_{\rm rms} = (1/2K)\sqrt{\pi W_F \cdot f/\zeta} \tag{23}$$

where W_F is the spectral density of the exciting forces given by

$$W_F = G^2{\rm rms}/\Delta f \tag{24}$$

with $G_{\rm rms}$ representing the root mean squarse value of the modal exciting force and Δf denoting the band width of the forcing frequency. In the case of octave band sound pressure levels with central frequency f_c, Δf is given by

$$\Delta f = f_c/\sqrt{2} \tag{25}$$

Equations (17) to (25) can be used to evaluate the probability of failure and other constraints. The feasible region, in which all the constraints are satisfied, is shown in the two-dimensional design space of Fig. 1. The contours of the objective function are also shown in the figure and the minimum weight can be seen to be 128.0 lb. The optimum values of the design variables are $h_{\rm opt} = 0.65''$ and $a_{\rm opt} = 50.0''$.

Conclusion

The problem of optimization of beam-like and plate-like structures with a constraint on the probability of failure due to combined acoustic and blast loading has been stated in the form of a standard nonlinear programing problem. The probability of failure has been estimated by approximating the response of the structure by that of a simple oscillator corresponding to the fundamental mode of the structure. The accuracy of the present analysis can be improved by approximating the structural behavior by several mode shapes. The number of design variables can be taken more than two and standard mathematical programing techniques can be applied to solve the problem.

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Liquid Drop Acceleration and Deformation

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Introduction

IT is well known[1-11] that when a liquid drop is subjected to rapid accelation by an air stream, lateral deformation and breakup are subsequent. The problems of aerodynamic stripping and catastrophic breakup are so overwhelming, that the simple dynamics of the liquid drop rely primarily on empirical correlation. In this Note, the dynamics are considered in the absence of the breakup and the time scales of the dynamics are shown to be slightly different than those used in empirical correlation.

To model the acceleration and lateral deformation of a liquid drop when subjected to a relative air stream, we extend an approach used by Reinecke and Waldman.[5] Writing the drop's momentum equation lateral to the relative gas velocity, Reinecke and Waldman use a moment method to develop an equation for the lateral deformation of the drop. Assuming that the lateral liquid velocity is linear in the lateral coordinate, they obtain

$$\Delta d^2\Delta/dT^2 + 2(d\Delta/dT)^2 = 4C_p \tag{1}$$

where C_p is the pressure coefficient at the raindrop stagnation point, Δ is the ratio of the drop diameter D to the initial diameter D_0.

$$\Delta = D/D_0$$

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and T is a nondimensional time

$$T = u_2 t \epsilon^{1/2} / D_0$$

where u_2 is the initial relative gas velocity, and ϵ is the ratio of gas density to liquid density.

The Reinecke and Waldman approach is limited to times for which the relative velocity u is approximately equal to its initial value u_2. Defining the dimensionless relative velocity $v = u/u_2$, we extend Eq. (1) to include a time dependent relative wind by replacing C_p with $C_p v^2$

$$\Delta d^2\Delta/dT^2 + 2(d\Delta/dT)^2 = 4\, C_p v^2 \quad (2)$$

and express the dynamics of v by

$$dv/dT = -(3C_D/4)\epsilon^{1/2}\Delta^2 v^2 \quad (3)$$

A simultaneous solution to Eqs. (2) and (3) is needed to determine the slowdown and lateral spreading of the deformable, incompressible drop.

Early and Late Time Solutions

To obtain a solution Eqs. (2) and (3), it is natural to seek a series expansion of the form

$$v = v_0 + \epsilon^{1/2} v_1 \text{----as } \epsilon \to 0$$

$$\Delta = \Delta_0 + \epsilon^{1/2}\Delta_1 \text{----as } \epsilon \to 0$$

where the lowest-order terms are the Reinecke-Waldman result.

$$\Delta_0 = (2C_p)^{1/2} T$$

$$v_0 = 1$$

Obtaining the next order term in v,

$$v = 1 - \epsilon^{1/2} C_D C_p T^3/2\text{----}$$

we note that the expansion is singular when both T and Δ are $0(\epsilon^{-1/6})$.

Rescaling the problem to "late time", we define

$$\tau = \epsilon^{1/6} T$$

$$\Gamma = \epsilon^{1/6}\Delta$$

and the Eqs. (2) and (3) become

$$\Gamma d^2\Gamma/d\tau^2 + 2(d\Gamma/d\tau)^2 = 4C_p v^2 \quad (4)$$

$$dv/d\tau = -(3C_D/4)v^2\Gamma^2 \quad (5)$$

The early time solution provides the initial conditions to Eqs. (4) and (5).

$$v\ (0) = 1 \quad (6)$$

$$\Gamma(0) = 0 \text{ as } \epsilon \to 0 \quad (7)$$

$$d\Gamma/d\tau = (2C_p)^{1/2} \quad (8)$$

Numerical solutions of Eqs. (4) and (5) for $\Gamma(\tau)$ and $v(\tau)$ are illustrated in Figs. 1 and 2, respectively. Theory is compared to the data of Reinecke and McKay.[4] The data illustrate that the drop does not deform as rapidly as theory predicts. This may be due to the aerodynamic stripping of the drop, but is also likely to be due to the lack of any retardation mechanism in the model. That is, once the pressure is applied

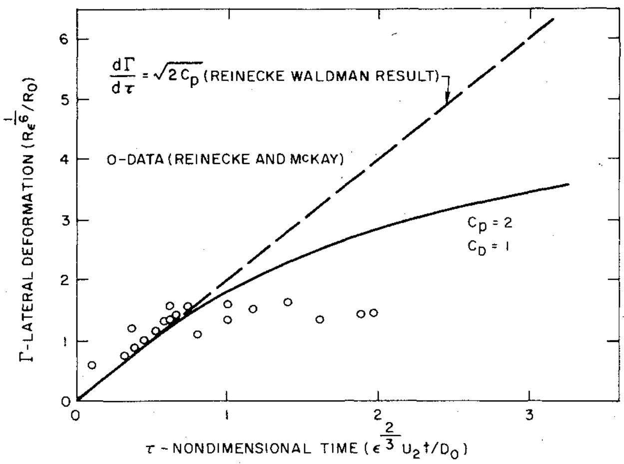

Fig. 1 Late time solution for the lateral deformation.

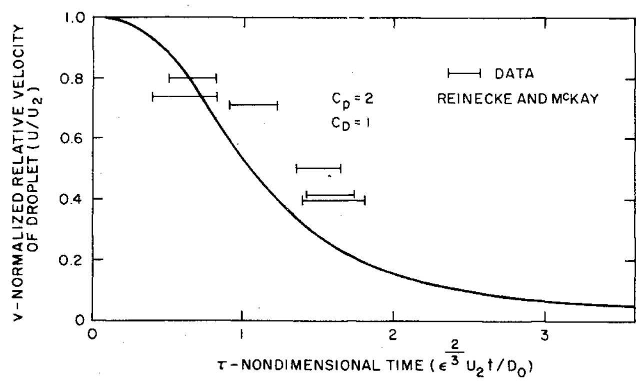

Fig. 2 Late time solution for the relative velocity.

and removed, the drop continues to deform. Figure 2 illustrates that the over prediction of the drop diameter results in an over prediction of the drop velocity (underprediction of relative wind velocity). Aerodynamic stripping can also affect the drop velocity directly through the mass loss terms in the momentum equation. It should be noted that this effect is also omitted in the present model.

The drop deformation causes the liquid drop to accelerate more rapidly than a rigid particle. If we consider the shock tube experiment where a stationary liquid drop is suddenly exposed to a relative wind u_2, the drop displacement (x) in the shock tube becomes

$$dx/dt = u_2 - u \quad (9)$$

In the "late time" variables, Eq. (9) becomes

$$dY/d\tau = 1 - v \quad (10)$$

where

$$Y = \epsilon^{1/3} x/D_0 \quad (11)$$

Equation (10) must be solved simultaneously with Eqs. (4) and (5). Making an expansion in τ, we obtain

$$\Gamma = (2C_p)^{1/2}\tau \quad (12)$$

$$v = 1 - \tfrac{1}{2} C_D C_p \tau^3 \quad (13)$$

and

$$Y = \tfrac{1}{8} C_D C_p \tau^4 \quad (14)$$

The time expansions are valid only for small τ. The solution at later time is obtained numerically and illustrated in Fig. 3. The results indicate that the τ dependence of Y decreases rapidly. A τ^4 dependence is valid at early time, decreasing to

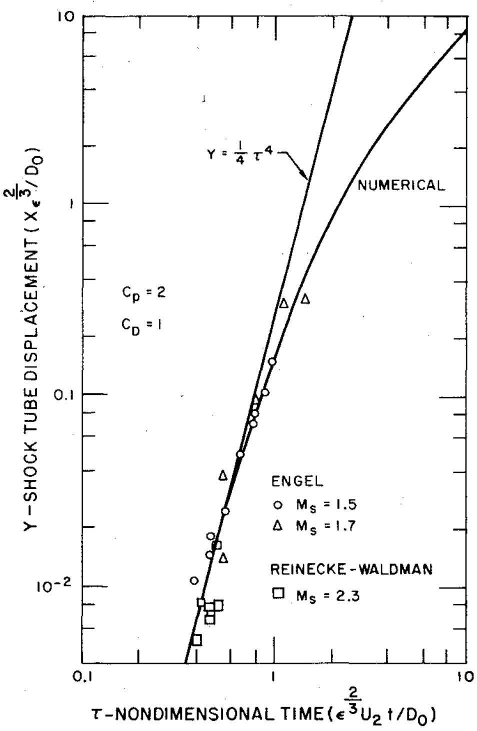

Fig. 3 Late time solution for the shock tube displacement.

τ^2 near $\tau=2$ and a linear behavior is noted for $\tau \geq 10$. These results appear to explain why investigators have been unable to correlate data with a single power law.

Conclusions

A simplified theory for liquid drop acceleration and deformation has been developed and compared to data. The results illustrate that the displacement increases as τ^4 at early time and as τ at late time. Hence, no single power law can correlate all of the data, although they may be good approximations over narrow ranges of τ.

Empirical results have previously been presented in nondimensional length and time scales which correspond to rigid particle acceleration. Reference variables for deformable particles have a different ϵ dependence than those for rigid particles. Hence, empirical results should be correlated in the reference variables appropriate to deformable particles.

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Large Amplitude Free Vibrations of Tapered Beams

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Nomenclature

a	= maximum amplitude
A	= area of cross-section of the beam at any x
A_c	= area of cross-section at $x=0$
b, d	= breadth and depth of the beam, respectively, at any x
b_c, d_c	= breadth and depth at $x=0$, respectively
b_e, d_e	= breadth and depth at $x=\pm l$, respectively
E	= Young's modulus
I	= moment of inertia at any x
I_c	= moment of inertia at $x=0$
l	= length of the beam
N_x	= axial force developed in the beam due to large deformation
r_c	= radius of gyration at $x=0$; given by $(I_c/A_c)^{1/2}$
u	= axial displacement
w	= transverse displacement
x	= axial coordinate measured from the center of the beam
α	= taper parameter; defined as $(b_c-b_e)/b_c$ for a beam with breadth taper and $(d_c-d_e)/d_c$ for a beam with depth taper
β	= a/r_c
ρ	= mass density
ξ	= nondimensional coordinate = $2x/l$
ω_{NL}, ω_L	= nonlinear and linear circular frequencies, respectively
$(\ )'$	= denotes differentiation with respect to the axial coordinate x
$(\dot{\ })$	= denotes differentiation with respect to time t

Introduction

LARGE amplitude free flexural vibrations of uniform-beams were studied, using continuum[1,2] and finite element methods.[3,4] Woinowsky-Krieger[1] obtained an elliptic integral solution, Srinivasan[2] used Ritz-Galerkin method, Chuh-Mei[3] and Venkateswara Rao et al.[4] used two different finite element formulations to obtain frequency-amplitude relationships for the fundamental mode.

In this Note, large amplitude vibrations of simply-supported and clamped tapered beams of rectangular cross-section, which are often encountered in practical structures, are investigated using Galerkin method. Two types of linear tapers, namely, breadth and depth tapers are considered in this study. For each type of taper, two solutions are obtained

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